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PRESERVATION TECHNOLOGY INVESTMENT POLICY – AN INVENTORY MODEL WITH STOCK-DEPENDENT DEMAND, TIME-VARYING HOLDING COST AND EXPONENTIAL PARTIAL BACKLOGGING

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Abstract

In this paper, an economic order quantity (EOQ) model is investigated for deteriorating items whose demand is influenced by stock. Deterioration rate is fixed and is controlled by using preservation technology. Inventory holding cost is a linear function of time. Shortage takes place at the end of the ordering cycle which is exponentially partially backordered in the next replenishment. Both the cases of partial backlogging and complete backlogging are studied and established with the help of suitable numerical examples. The purpose of this study is to obtain optimal replenishment and preservation technology investment policy together with optimal cycle length and shortage period by minimizing the average system cost.

Keywords:

Stock-dependent demand;

Time varying holding cost;

Deterioration;

Preservation technology;

Backlogging.

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1. Introduction

In general, almost all items deteriorate more or less. So, deterioration of inventory is an important issue for any business. The rate of deterioration can be controlled with the help of preservation technology. Several research works have been incorporated to control deterioration of product by using preservation technology. Hsu *et al*. [1] have studied an inventory model where they have assumed that the retailer invests a preservation technology to reduce the deterioration rate of items. The effect of preservation technology investment on deteriorating inventory together with partial backlogging is discussed by Lee and Dye [2] and Mishra [3]. Hsiesh and Dye [4] have determined optimal production and preservation technology investment strategies for a production-inventory model while a joint pricing; service and preservation technology investment policy for deteriorating product under common resource constraints is presented by Zhang *et al*. [5].

In classical inventory studies, the EOQ models are framed for fixed demand. But in reality demand may not be constant. Generally, a large stock display in supermalls encourages customers to buy more and more products. Several researchers have studied inventory models for stock-dependent demand rate. Sarker *et al*. [6] have presented an order level lot size inventory model for deteriorating products where the demand is assumed to be stock-dependent. Hwang and Hahn [7] have determined an optimal procurement policy for the items having fixed life time and inventory level dependent consumption rate whereas Hou [8] has incorporated a deteriorating inventory model for stock dependent demand together with inflation and time discounting. Yang *et al*. [9] have investigated an inventory model for deteriorating items with inventory level dependent demand in conjunction with partial backlogging and inflation while a markdown policy is addressed by Das Roy [10] for deteriorating items having stock and sales price sensitive demand.

Stock-out situation is a very common phenomenon in businesses. Many researchers have included the occurrence of shortages in their studies [Wu *et al*. [11], Chen and Lo [12], Das Roy and Sana [13], Das Roy [14]]. In case of deteriorating or perishable inventory it is almost certain to take place. To maintain goodwill with customer backlogging is one of the useful tricks of a business. Several research works have been done by considering partial and complete backlogging. Some of them are Min and Zhou [16], Das Roy *et al*. [17] and Dutta and Kumar [18].

This paper analyses an inventory model for deteriorating products whose demand is considered to be stock dependent. The rate of deterioration is constant. Preservation technology is taken into consideration to reduce the deterioration rate of items. Shortages occur and back ordered. An exponential partial backlogging is considered. Both types of backlogging, partial and complete, are discussed separately. Demand during shortage period is assumed to be constant. The objectiveof this study is to minimize the average system cost and determine the optimal values of the ordered lot size, shortage period, cycle length, preservation technology investment and the optimum average system cost.

The whole article consists of 6 Sections. The introduction part is given in Section 1 whereas Section 2 contains the notation and assumptions of the model. Section 3 describes the mathematical formulation while the solution procedure is discussed in Section 4. Section 5 provides numerical analysis. The conclusion of the study with further research scope is given in Section 7.

2. Notation and assumptions

The notations and assumptions used to construct the model are as follows.

2.1. Notation

C_0 : Ordering cost per cycle.

C_P : Unit purchasing cost per unit time.

$H(t)$: Unit holding cost per unit time

C_D : Unit deterioration cost per unit time.

C_B : Unit back ordering cost per unit time.

C_L : Unit lost sale cost per unit time.

θ : Rate of deterioration.

ξ : Preservation technology (PT) cost for reducing deterioration rate (decision variable).

$m(\xi)$: The reduced deterioration rate due to the use of Preservation technology (PT).

r : Percentage of demand backlogged.

t_1 : The time point at which the stock reaches to zero, $t_1 \geq 0$ (decision variable).

t_2 : Shortage time, $t_2 \geq 0$ (decision variable).

T : Length of ordering cycle (decision variable)..

q_i : Inventory level at time t, $0 \leq t \leq t_1$

q_s : Inventory level at time t,$t_1 \leq t \leq t_1 + t_2$.

$K(q_i)$: Demand of items at time $t \geq 0$.

S_M : The maximum level of on-hand inventory

S_B : The maximum backlogging level during the stock-out period $[t_1, t_1 + t_2]$.

Q : Order lot size per cycle (decision variable).

A_P : The average system costfor Case I.

A_C : The average system cost for Case II.

2.2. Assumptions

1. The EOQmodel is discussed for deteriorating inventory.
2. Demand depends on stock i.e., $K(q_i) = \mu + \omega q_i$, $\mu > 0$; $\omega > 0$, whereμindicates the initial demand, ωindicates the rate at which the demand rate changes with inventory level.

3. Rate of deterioration is constant.
4. Preservation technology is used to control deterioration rate of products.
5. The reduction of the deterioration rate $m(\xi)$ is as follows

$$m(\xi) = \theta(1 - e^{-\alpha\xi}), \quad \alpha > 0.$$

6. There is no repair or replacement of deteriorating product during the time interval taken into consideration.
7. Shortage takes place and partially backlogged in the next replenishment.
8. Inventory holding cost is considered to be a linear function of time i.e.,

$$H(t) = H_0 + H_1 t \ , \ H_0 > 0; \ H_1 > 0.$$

9. Replenish rate is infinite.

$$H(t) = H_0 + H_1 t \ , \ H_0 > 0; \ H_1 > 0.$$

3. Mathematical Formulation

The ordered lot size is Q. The cycle begins at time $t = 0$ when the on-hand inventory is S_M after clearing the previous shortages. Deterioration takes place at the start of the ordering cycle. Inventory depletes due to demand and deterioration and becomes zero at time $t = t_1$. Shortage occurs during $[t_1, t_1 + t_2]$. The demand of the stock-out period is exponentially partially backordered [Das Roy *et al*. [19], [20]]. The inventory model is shown in Figure 1.

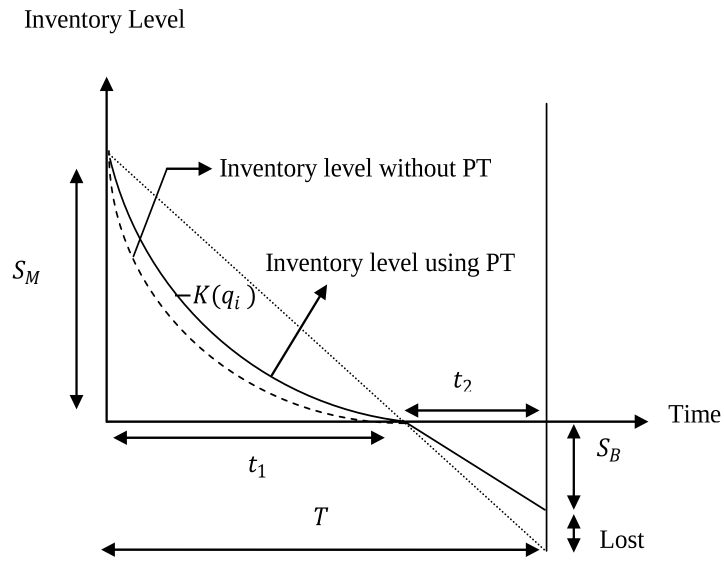

Figure 1. *Inventory versus time*

The differential equations of the inventory system at time *t* are

$$q_i{}'(t) + (\theta - m(\xi))q_i(t) = -(\mu + \omega q_i),\ 0 \le t \le t_1, \qquad q_i(t_1) = 0 \tag{1}$$

and

$$q_s{}'(t) = -\mu e^{-\delta(t_1+t_2-t)},\ t_1 \le t \le t_1+t_2, \quad \delta > 0, \quad q_s(t_1) = 0. \tag{2}$$

Using the boundary conditions the solutions of equations (1) and (2) becomes

$$q_i(t) = \frac{\mu}{(\theta - m(\xi) + \omega)}\left(e^{(\theta - m(\xi)+\omega)(t_1 - t)} - 1\right), \qquad 0 \le t \le t_1 \tag{3}$$

and $$q_s(t) = -\frac{\mu}{\delta}\left(e^{-\delta(t_1+t_2-t)} - e^{-\delta t_2}\right), \qquad t_1 \le t \le t_1 + t_2.$$ (4)

The maximum inventory level at time $t = 0$ is as follows.

$$S_M = q_i(0) = \frac{\mu}{(\theta - m(\xi) + \omega)}\left(e^{(\theta-m(\xi)+\omega)t_1} - 1\right).$$

The maximum backlogging inventory level S_B is

$$S_B = -q_s(t_1 + t_2) = \frac{\mu}{\delta}\left(1 - e^{-\delta t_2}\right).$$

Ordered lot size $$Q = S_M + S_B = \frac{\mu}{(\theta-m(\xi)+\omega)}\left(e^{(\theta-m(\xi)+\omega)t_1} - 1\right) + \frac{\mu}{\delta}\left(1 - e^{-\delta t_2}\right).$$ (5)

The length of ordering cycle $T = t_1 + t_2$. (6)

Now the inventory related costs per cycle are as follows.

Ordering cost $OC = C_0$.

Purchasing cost $PC = C_P Q$.

Holding cost$HC = C_H \int_0^{t_1} H(t) q_i(t)\, dt$

$$= \frac{\mu}{(\theta - m(\xi) + \omega)}\left[\left(H_0 + \frac{H_1}{(\theta - m(\xi) + \omega)}\right)\left(\frac{e^{(\theta-m(\xi)+\omega)t_1} - 1}{(\theta - m(\xi) + \omega)} - t_1\right) - \frac{1}{2} H_1 {t_1}^2\right].$$

Deterioration cost $DC = C_D \times (\theta - m(\xi)) \times \int_0^{t_1} q_i(t)\, dt$

$$= \frac{C_D(\theta - m(\xi))\mu}{(\theta - m(\xi) + \omega)}\left(\frac{e^{(\theta-m(\xi)+\omega)t_1} - 1}{(\theta - m(\xi) + \omega)} - t_1\right).$$

Preservation Technology cost $PTC = \xi T$.

3.1 Case I: Partial Backlogging

In this case, it is assumed that a portion of the unsatisfied demand of the stock-out period is satisfied in the next replenishment and the remaining portion is lost forever.

Backlogging cost $BC = C_B \int_{t_1}^{t_1+t_2} (-q_s(t))\, dt = \frac{C_B \mu}{\delta^2}\left(1 - e^{-\delta t_2} - \delta t_2 e^{-\delta t_2}\right)$.

Lost sale cost $LC = C_L \int_{t_1}^{t_1+t_2} \mu(1-r)\, dt = \frac{C_L \mu}{\delta}\left(\delta t_2 - 1 + e^{-\delta t_2}\right)$.

Then the average system cost in this case is

$$A_P(t_1, t_2) = \frac{1}{T}[OC + PTC + PC + HC + DC + BC + LC]$$

$$= \frac{1}{T}\Bigg[C_0 + \xi + C_P Q + \frac{\mu}{(\theta - m(\xi) + \omega)} \times \left\{\left(H_0 + C_D\big(\theta - m(\xi)\big) + \frac{H_1}{(\theta - m(\xi) + \omega)}\right)\left(\frac{e^{(\theta - m(\xi)+\omega)t_1} - 1}{(\theta - m(\xi) + \omega)} - t_1\right) - \frac{1}{2} H_1 {t_1}^2\right\} + \frac{C_B \mu}{\delta^2}\left(1 - e^{-\delta t_2} - \delta t_2 e^{-\delta t_2}\right) + \frac{C_L \mu}{\delta}\left(\delta t_2 - 1 + e^{-\delta t_2}\right)\Bigg]. \quad (7)$$

3.2 Case II: Complete Backlogging

Suppose the shortage is very small i.e. $\delta t_2 << 1$. Neglecting the terms higher than first degree in δt_2. Then the maximum backlogging inventory level is

$$S_B = \mu t_2.$$

Ordered lot size $Q = S_M + S_B = \frac{\mu}{(\theta - m(\xi)+\omega)}\left(e^{(\theta - m(\xi)+\omega)t_1} - 1\right) + \mu t_2$.

(8)

Backlogging cost $BC = C_B \mu {t_2}^2$.

Lost sale cost $LC = 0$.

So, no lost sale takes place. This is the case of complete backlogging.

The average system cost of this case is

$$A_C(t_1, t_2) = \frac{1}{T}\Bigg[C_0 + \xi + C_P Q + \frac{\mu}{(\theta - m(\xi) + \omega)} \times \left\{\left(H_0 + C_D\big(\theta - m(\xi)\big) + \frac{H_1}{(\theta - m(\xi) + \omega)}\right)\left(\frac{e^{(\theta - m(\xi) + \omega)t_1} - 1}{(\theta - m(\xi) + \omega)} - t_1\right) - \frac{1}{2} H_1 {t_1}^2\right\} + C_B \mu {t_2}^2\Bigg] \qquad (9)$$

4. Solution procedure

The expression in average system cost function is a complex one. “MATHEMATICA 8.0" software is used to obtain the solutions and examine the following optimization criteria for both Case I and Case II.

For Case I: $\frac{\partial^2 A_P}{\partial \xi^2} > 0$, $\frac{\partial^2 A_P}{\partial {t_1}^2} > 0$, $\frac{\partial^2 A_P}{\partial {t_2}^2} > 0$,

and the hessian matrix $H = \begin{pmatrix} \frac{\partial^2 A_P}{\partial \xi^2} & \frac{\partial^2 A_P}{\partial \xi \partial t_1} & \frac{\partial^2 A_P}{\partial \xi \partial t_2} \\ \frac{\partial^2 A_P}{\partial t_1 \partial \xi} & \frac{\partial^2 A_P}{\partial {t_1}^2} & \frac{\partial^2 A_P}{\partial t_1 \partial t_2} \\ \frac{\partial^2 A_P}{\partial t_2 \partial \xi} & \frac{\partial^2 A_P}{\partial t_2 \partial t_1} & \frac{\partial^2 A_P}{\partial {t_2}^2} \end{pmatrix}$ is positive definite.

For Case II: $\frac{\partial^2 A_C}{\partial \xi^2} > 0$, $\frac{\partial^2 A_C}{\partial {t_1}^2} > 0$, $\frac{\partial^2 A_C}{\partial {t_2}^2} > 0$, and the hessian matrix $H = \begin{pmatrix} \frac{\partial^2 A_C}{\partial \xi^2} & \frac{\partial^2 A_C}{\partial \xi \partial t_1} & \frac{\partial^2 A_C}{\partial \xi \partial t_2} \\ \frac{\partial^2 A_C}{\partial t_1 \partial \xi} & \frac{\partial^2 A_C}{\partial {t_1}^2} & \frac{\partial^2 A_C}{\partial t_1 \partial t_2} \\ \frac{\partial^2 A_C}{\partial t_2 \partial \xi} & \frac{\partial^2 A_C}{\partial t_2 \partial t_1} & \frac{\partial^2 A_C}{\partial {t_2}^2} \end{pmatrix}$ is positive definite.

5. Numerical Analysis

Example 1. The values of the parameters related to the model described in Case I in appropriate units are as follows

$C_0 = \$120$, $C_P = \$4$, $H_0 = \$0.2$, $H_1 = \$0.1$, $C_D = \$0.3$, $C_B = \$1.5$,
$C_L = \$2.2$, $\mu = 220$, $\omega = 0.01$, $\theta = 0.3$, $r = 0.8$, $\delta = 0.1$.

Using the above parameter values in equation (7) the optimal solution obtained for Case I are :${t_1}^* = 1.7496 \cong 1.75$ units,${t_2}^* = 0.549205 \cong 0.55$ unit, $\xi^* = \$62.7391 \cong \$62.74$ and ${A_P}^* = \$1030.39$. Substituting the values of ${t_1}^*$ and ${t_2}^*$ in equation (5) and (6) the optimal values of ordered lot size and cycle length are determined as $Q^* = 510.337 \cong 510$ units and $T^* = 2.3$ units respectively.

Example 2. All the values of the parameters are assumed to be same as Example 1 then the optimal solutions for Case II are obtained from equation (9) as: ${t_1}^* = 1.88774 \cong 1.89$ units${t_2}^* = 0.245067 \cong 0.24$ unit, $\xi^* = \$65.8335 \cong \$65.83$ and${A_C}^* = \$1041.74$. Using these values of ${t_1}^*$ and ${t_2}^*$ in equation (8) and (6) the optimal ordered lot size and cycle length are obtained as $Q^* = 477.026 \cong 477$ units and $T^* = 2.13$ units respectively.

6. Conclusion

In this study, a deteriorating inventory model is discussed for inventory level dependent demand with shortages. Rate of deterioration is fixed. Emphasis is given on reduction of deterioration rate by using preservation technology. It is a common practice to assume inventory holding cost as a constant but in reality stock holding cost may also vary with time, especially for deteriorating products which needs extra care. Another realistic assumption is backlogging. Generally, if shortage takes place the seller wishes to fulfill the unsatisfied demand of the customer in the next replenishment to minimize his loss and keep goodwill. Sometimes he is capable to do it completely; sometimes partially. Here, both of the cases partial back ordering and complete back ordering are discussed. Using same set of parameter values in numerical example it is noted that the average system cost for the case of complete backlogging is greater than the case of partial backlogging which is obvious.

This model is developed for stock-dependent demand and constant deterioration rate. Some other types of demand pattern and deterioration rate may be considered for further study.

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